Prove that $\frac{1}{1\times 6}+\frac{1}{6\times 11}+\frac{1}{11\times 16}+\cdots+\frac{1}{(5n-4)\times(5n+1)}=\frac{n}{5n+1}$ for all integers $n\geq 1$ **SCORE: _____ / 10 PTS**

by mathematical induction, showing all steps demonstrated in lecture.

BASIS STEP:

If $n=1$, $\boxed{\frac{1}{1\times 6}=\frac{1}{6} \qquad \frac{1}{5(1)+1}=\frac{1}{6}}$ 1

INDUCTIVE STEP:

1 $\boxed{\text{Assume } \frac{1}{1\times 6}+\frac{1}{6\times 11}+\frac{1}{11\times 16}+\cdots+\frac{1}{(5k-4)\times(5k+1)}=\frac{k}{5k+1}}$ $\boxed{\text{for some arbitrary but particular integer } k}$ 1

2 $\boxed{\frac{1}{1\times 6}+\frac{1}{6\times 11}+\frac{1}{11\times 16}+\cdots+\frac{1}{(5k-4)\times(5k+1)}+\frac{1}{(5k+1)\times(5k+6)}}$

1 $\boxed{=\frac{k}{5k+1}+\frac{1}{(5k+1)(5k+6)}}$

FIRST & LAST 2 TERMS MUST BOTH BE SHOWN

$=\frac{k(5k+6)+1}{(5k+1)(5k+6)}$

1 $\boxed{=\frac{5k^2+6k+1}{(5k+1)(5k+6)}}$

1 $\boxed{=\frac{(5k+1)(k+1)}{(5k+1)(5k+6)}}$

$=\frac{k+1}{5k+6}$

1 $\boxed{=\frac{k+1}{5(k+1)+1}}$

1 $\boxed{\text{By mathematical induction, } \frac{1}{1\times 6}+\frac{1}{6\times 11}+\frac{1}{11\times 16}+\cdots+\frac{1}{(5n-4)\times(5n+1)}=\frac{n}{5n+1} \text{ for all integers } n\geq 1}$

Evaluate $\sum_{k=1}^{200}(8k-3k^2)$. **Your final answer must be a number (not involving arithmetic operations).** SCORE: _____ / 4 PTS

$$= \underline{8\sum_{k=1}^{200} k - 3\sum_{k=1}^{200} k^2} \ 1$$

$$= 8 \cdot \underline{\tfrac{1}{2}(200)(201)} \ 1 - 3 \cdot \underline{\tfrac{1}{6}(200)(201)(401)} \ 1$$

$$= \underline{-7899300} \ 1$$

Find the 7^{th} term of $(11b-8g)^{26}$. **Your final coefficient may be in factored form as shown in lecture.** SCORE: _____ / 5 PTS

$${}_{26}C_6 (11b)^{26-6}(-8g)^6$$

$$= \frac{26!}{6!\,20!}(11b)^{20}(-8g)^6 \quad \left(\tfrac{1}{2}\right) \quad \left(\tfrac{1}{2}\right)$$

PLUS $\left(\tfrac{1}{2}\right)$ IF YOUR FINAL ANSWER IS POSITIVE

$$= \boxed{\frac{26\cdot 25\cdot 24\cdot 23\cdot 22\cdot 21\cdot 20!}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 20!}}\, 11^{20}\, 8^6\, b^{20} g^6 \quad (1)$$

$$= 26\cdot 5\cdot 23\cdot 11\cdot 7\cdot 11^{20}\cdot 8^6 b^{20} g^6 = 230230\cdot 11^{20}\cdot 8^6\, b^{20} g^6$$

$\left(\tfrac{1}{2}\right)$ $\left(\tfrac{1}{2}\right)$ $\left(\tfrac{1}{2}\right)$ (1)

If $f(x)=x^5$, expand and **completely simplify** the difference quotient $\dfrac{f(x+h)-f(x)}{h}$. SCORE: _____ / 5 PTS

$$\frac{(x+h)^5-x^5}{h} = \frac{\cancel{x^5}+5x^4h+10x^3h^2+10x^2h^3+5xh^4+h^5-\cancel{x^5}}{h} \quad (3)$$

$$= 5x^4+10x^3h+10x^2h^2+5xh^3+h^4 \quad (2)$$

Expand and **completely simplify** the complex number $(3-2i)^4$. SCORE: _____ / 6 PTS

$$1(3)^4(-2i)^0 + 4(3)^3(-2i)^1 + 6(3)^2(-2i)^2 + 4(3)^1(-2i)^3 + 1(3)^0(-2i)^4 \quad \left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)$$

$$= 81 + 4(27)(-2i) + 6(9)(-4) + 4(3)(8i) + 16 \quad \left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)$$

$$= 81 - 216i - 216 + 96i + 16 \quad \left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)(1)$$

$$= -119-120i \quad \left(1\tfrac{1}{2}\right)$$